φ as functions of time in order that the torques can be used in the attitude control equations. This problem will be solved for a circular orbit. Consider two sets of Cartesian coordinates, one (x,y,z) fixed in the earth, and one (x_0,y_0,z_0) fixed with respect to the orbital plane. Without loss of generality, it can be assumed that the orbital plane contains the x_0 axis. In the (x_0,y_0,z) system of coordinates,

$$z = r\cos\delta\cos\omega_1 t \quad (7)$$

$$x_0 = r\sin\omega_1 t \quad (8)$$

$$y_0 = r\sin\delta\cos\omega_1 t \quad (9)$$

where δ is the angle that the orbital plane makes with the z axis and ω_1 is the frequency of the satellite orbit. Transforming to the x,y,z coordinates by means of

$$x = x_0\cos\omega_2 t + y_0\sin\omega_2 t \quad (10)$$

$$y = -x_0\sin\omega_2 t + y_0\cos\omega_2 t \quad (11)$$

where ω_2 is the frequency of the earth's rotation, one obtains

$$\theta = \cos^{-1}(z/r) = \cos^{-1}[\cos\delta\cos\omega_1 t] \quad (12)$$

$$\varphi = \tan^{-1}(y/x) = \tan^{-1}\left[\frac{-\sin\omega_1 t\sin\omega_2 t + \sin\delta\cos\omega_1 t\cos\omega_2 t}{\sin\omega_1 t\cos\omega_2 t + \sin\delta\cos\omega_1 t\sin\omega_2 t}\right] \quad (13)$$

The angle between the roll axis and the meridians of longitude is given by

$$\gamma = \tan^{-1}(\tan\delta\csc\omega_1 t) \quad (14)$$

Equations (1–3 and 12–14), when substituted into Eqs. (4–6), will give the electromagnetic torques operating on the satellite as explicit functions of time. For a polar ($\delta = 0$) orbit, Eqs. (12–14) reduce to $\theta = \omega_1 t$, $\varphi = -\omega_2 t$, and $\gamma = 0$.

Present designs of satellites bearing Snap reactor power systems call for a cylindrical configuration having approximately equal moments of inertia about pitch and roll axes, and a much smaller moment about the yaw axis. In this case, it is necessary to stabilize the satellite about the yaw axis with some arrangement of gyroscopes, and it is therefore desirable to minimize the magnetic torques about the yaw axis.

Since the azimuthal component of the geomagnetic field B_φ is generally small when compared with B_θ or B_r, Eq. (6) shows that T_Y can be minimized by locating the satellite in a polar orbit ($\gamma = 0$) and by aligning the satellite magnetic moment along the yaw axis. If this latter requirement cannot be accomplished, in no case should the satellite moments be aligned with the pitch axis.

Large-Scale Gap Test: Comparison of Tetryl and Pentolite Donors

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In the large-scale shock sensitivity test, it is assumed that the pressure amplitude at the 50% point is an intrinsic property of a propellant tested under standardized conditions, provided that the shape of the pressure pulse is defined by the amplitude. In order to examine this proviso and to study the adequacy of pressure amplitude as a measure of shock sensitivity, a second calibration for the gap test was made with a pentolite donor replacing the tetryl donor of the standardized test.

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The large-scale shock sensitivity test (gap test) was calibrated originally at this laboratory with a tetryl donor[1, 2] to interpret the 50% point gap in terms of absolute pressure. The pressure amplitude at the 50% point, assuming the shape of the pressure pulse to be defined by the amplitude, should be an intrinsic property of a propellant tested under standardized conditions and should be reproducible regardless of the donor used. To study the adequacy of 50% pressure as measure of shock sensitivity, a standard pentolite donor was made and used in a second calibration. This donor also was used to determine the 50% point of various substances; the pressures obtained at the 50% point were compared to those obtained with the standard tetryl donor.

Experimental Method

The ingredients of pentolite, 50% trinitrotoluene (TNT) and 50% pentaerythrite tetranitrate (PETN), were prepared according to the joint Army-Navy specification.[3, 4] In addition, careful control of the particle size, mixing process, and density of the pellets was exercised. A no. 70 and a no. 100 sieve (U. S. Standard Sieve Series—ASTM specification) were used to obtain particle sizes of the PETN and TNT ranging from 150 to 210 μ. One thousand grams of each ingredient were then added to a V-blender and dry blended for 1 hr to insure a homogeneous mixture. The pentolite

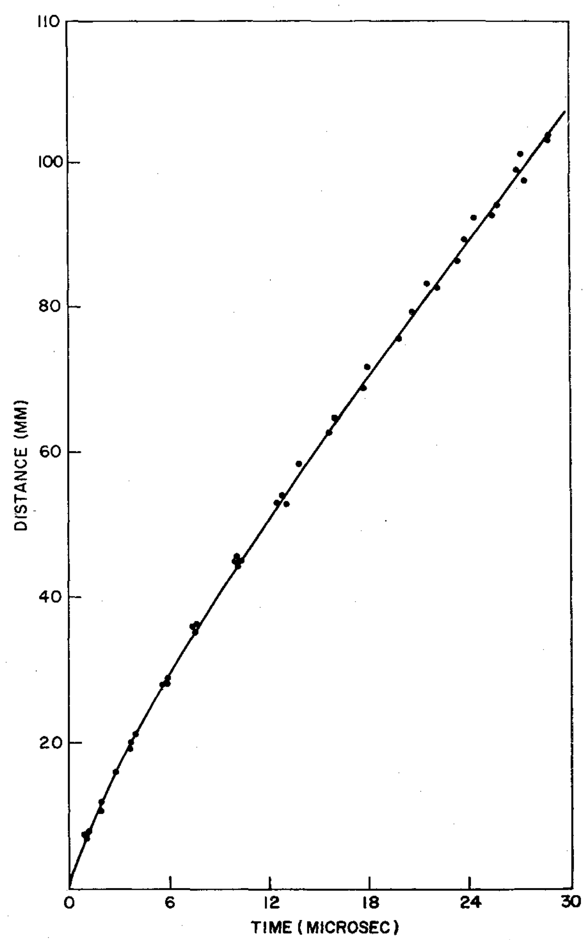

Fig. 1 Shock in Plexiglas

Table 1 Distance vs time in Plexiglas rod

Experiment 1		Experiment 2		Experiment 3		Experiment 4	
Time, μsec	Distance, mm	Time, μsec	Distance, mm	Time, μsec	Distance, mm	Time, μsec	Distance, mm
1.21	7.2	1.87	10.4	1.31	8.0	1.18	7.2
3.83	20.0	3.76	18.9	2.89	16.0	2.01	11.5
5.69	28.0	5.82	28.1	5.85	28.9	4.08	21.2
7.71	35.7	7.70	35.3	7.75	35.9	7.64	36.0
10.26	44.5	10.36	45.0	10.19	44.3	10.19	45.5
12.68	52.4	12.84	53.6	13.14	53.1	13.88	58.0
15.73	62.0	16.07	64.2	17.69	68.5	18.07	71.3
23.40	85.7	20.74	79.0	19.99	75.5	21.63	82.8
25.53	92.2	23.84	89.0	22.23	82.3	24.53	92.0
27.43	97.7	26.96	98.6	25.78	93.6	27.34	100.5
28.87	102.2			28.86	102.8		

thus formed was placed into a mold of 2-in. i.d. and hydraulically pressed to a length of 1 ± 0.003 in. and to a density between 1.56 and 1.57 g/cm³, 91 to 92% of the maximum theoretical density (TMD), 1.71 g/cm³.

The experimental procedure for shock wave velocity measurement was essentially that of the previous work[1] and duplicates the conditions of the shock sensitivity test.[2] Distance-time data for the four experiments run are given in Table 1 and plotted in Fig. 1.

Velocity determinations for the first 10 to 15 mm of the gap were difficult to make graphically. A slight variation in the interpretation of the data in this region generates a rather large variation in the calculated pressure. The error may be increased further by inaccurate slope measurements or by errors inherent in the equation of state used to calculate shock pressure.

To obtain the best interpretation, a number of equations ranging from a second- to a seventh-degree polynomial were fitted to the experimental data by an electronic computer (IBM-7090). A fifth-degree equation reproduced the experimental data to a fair degree of accuracy. The first derivatives of the equation

$$X = 0.377 + 5.903t - 0.250t^2 + 0.021t^3 - 0.297 \times 10^{-3}t^4 + 0.281 \times 10^{-5}t^5$$

where

X = distance, mm
t = time, μsec

were used to obtain the shock velocities.

Table 2 lists the data for pressure and shock velocity vs distance. The plot of shock pressure vs distance for pentolite and tetryl[1] is shown in Fig. 2.

Discussion

The gap used in the Naval Ordnance Laboratory shock sensitivity test is composed of Plexiglas, Lucite, cellulose acetate, or some combination of these materials. These substances are quite similar, and it has been demonstrated[1] that they are equivalent as attenuators in the gap test. Figure 2 shows the relationship between pressure and distance (gap) for both pentolite and tetryl. Both donors were calibrated under similar conditions with one exception: the Plexiglas rod used here was slightly smaller, $1\frac{7}{8}$ in. between the flat parallel surfaces as against 2 in. in the earlier work with tetryl. This should not affect the results obtained to any noticeable degree. The same equation of state for the attenuator was used in both calibrations to calculate the pressure-distance relationship for the gap.

Similar work using cast pentolite 1.5 in. in diameter by 0.8-in. long has been done by Cook and Udy.[5] Their pressure vs distance curve shows values of pressure consistently lower than those reported here but exhibiting the same general shape. Quantitative differences are to be expected, since Cook and Udy used cast rather than pressed pentolite in geometries different from the present work. Analogous differences appear in their calibration with tetryl for which the geometry differed from that for the pentolite as well as from that of the present work.

From Fig. 2 one sees that at zero gap the shock pressure of the pentolite donor is somewhat larger than that of the tetryl. This larger pressure is attenuated rapidly. After 10 mm it is within the tetryl pressure range, and after 25 mm (1 in.) of travel its curve approximates that of the tetryl. From this point on, both donors may be considered to give the same pressure amplitude within the precision of the experimental data.

The pressure amplitude at the 50% point as a quantitative measure of sensitivity then was studied by making a series of shock sensitivity tests on several different materials. A number of charges were made from the same batch of materials, and the 50% point gap was determined using first a tetryl donor and then a pentolite donor. The results are listed in Table 3.

For gaps larger than 50 cards (13 mm), the pressure amplitude for the same substance measured by the tetryl system and the pentolite system differ by ±5% from the mean. The values for gaps less than 50 cards differ by ±13 to 20%, with increasing difference for decreasing gap length (see Table 3).

It can be concluded that the same initiating pressure (to within 5%) is measured by either donor at large gaps. For

Table 2 Pressure and shock velocity as a function of distance

Distance, mm	Shock velocity, mm/μsec	Pressure, kbar
2	5.82	146.1
5	5.52	125.9
7	5.33	114.1
10	5.08	99.1
12	4.92	89.5
14	4.76	80.9
16	4.63	73.6
20	4.38	61.3
24	4.16	51.2
28	3.98	43.0
32	3.82	36.7
40	3.57	27.5
48	3.37	20.6
60	3.18	14.6
70	3.06	11.0
80	2.97	8.9
90	2.90	7.3
100	2.84	5.7

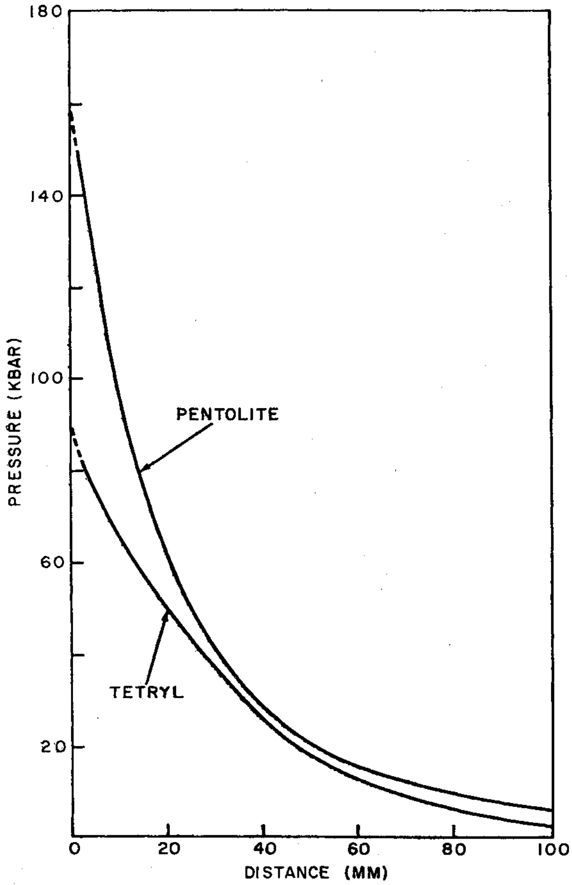

Fig. 2 Pressure vs gap

Table 3 Pentolite vs tetryl: shock sensitivity

Material	Donor	Gap, 50% point	Pressure, kbar	Mean, kbar
comp. B-3 (cast)	tetryl	209	16.4	
	pentolite	209	18.0	17.2
nitroguanidine	tetryl	46	63.0	
= 1.59 g/cm³	pentolite	53	83.2	73.1
nitroguanidine/wax	tetryl	16	78.8	
95/5	pentolite	25	119.7	99.3
= 1.55 g/cm³				

smaller gaps, agreement between the donors is not obtained because the calibration curves in this region are inaccurate, or because the pressure-time loading curves (not measured) affect the result, or because both of these factors are operative. As one approaches zero gap the pressure-time histories of the two donors should differ, and this factor probably has a major effect on inducing detonation of the acceptor. In other words, at the highest pressures, pressure amplitude alone does not define the shock sufficiently.

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Calculation of Damped Linear Systems by Holzer's Method

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HOLZER's tabular method for the calculation of torsional and other discrete linear systems is very well known. When treating the free vibrations by this method, i.e., when evaluating the natural frequencies, it generally is stated in the literature that the parameter of the Holzer table is the frequency ω.

The fact is, however, that in the case of free vibrations the parameter of the Holzer table is the eigenvalue Λ rather than the frequency ω.

To substantiate this statement, consider first the standard single-degree-of-freedom system that is damped both externally and internally. In this system, $(-s\dot{\varphi})$ = external damping torque (damping on the mass), and $(-u\dot{\varphi})$ = internal damping torque (damping parallel to the spring).

The differential equation is

$$-J\ddot{\varphi} - s\dot{\varphi} - u\dot{\varphi} - k\varphi = 0 \tag{1}$$

Substitution of the solution assumption $\varphi = \Phi e^{\Lambda t}$ in Eq. (1) gives the eigenvalue $\Lambda_{1,2}$ in the form

$$\Lambda_{1,2} = -[(s+u)/2J] \pm j\{\omega_v^2 - [(s+u)/2J]^2\}^{1/2} \qquad j = (-1)^{1/2} \tag{2}$$

hence in the form $\Lambda_{1,2} = -h \pm j\omega_D$. ($\omega_D$ = damped natural frequency; ω_v = undamped natural frequency.)

Generalizing this for the n mass system, one can say that, in the case of the solution assumption $\varphi = \Phi e^{\Lambda t}$, the eigenvalues of a linear system are of the form ${}_m\Lambda_{1,2} = -({}_mh) \pm j({}_m\omega_D)$. (Subscript m refers to the mth mode.)

Now, consider a multimass system damped both externally and internally. If the system is undamped ($s_i = u_i = 0$), the differential equation for the first mass reads

$$-J_1\ddot{\varphi}_1 - k_1(\varphi_1 - \varphi_2) = 0 \tag{3}$$

For the undamped system the "one-phase" solution assumption $\varphi_i = \Phi_i \sin\omega t$ is acceptable because there is no phase shift between the masses. Substituting it in Eq. (3), one has

$$\Phi_2 = \Phi_1 - (J_1\omega^2/k_1)\Phi_1 \tag{4}$$

which, as is well known, is the relation upon which the *ordinary* Holzer table for the undamped system is based.

If the multimass system considered is damped, then the differential equation for the first mass reads

$$-J_1\ddot{\varphi}_1 - s_1\dot{\varphi}_1 - u_1(\dot{\varphi}_1 - \dot{\varphi}_2) - k_1(\varphi_1 - \varphi_2) = 0 \tag{5}$$

Now a "two-phase" solution, hence either $\varphi_i = \Phi_i e^{\Lambda t}$ or $\varphi_i = e^{j\Lambda t}$, must be accepted because the damping produces a phase shift.

Upon substituting the solution assumption $\varphi_i = \Phi_i e^{\Lambda t}$ in Eq. (5), one obtains

$$\Phi_2 = \Phi_1 - [(-J_1\Lambda^2 - s_1\Lambda)/(k_1 + u_1\Lambda)]\Phi_1 \tag{6}$$

Comparing Eqs. (4) and (6), one sees that $J_1\omega^2$ and k_1 with the undamped system correspond to $-J_1\Lambda^2 - s_1\Lambda$ and $k_1 +$

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